

Effects of Disorder on the Frequency and Field of Photonic Crystal Cavity Resonators

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In recent years the application of 2-Dimensional (2D) metallic Photonic Crystal (PC) structures to high power microwave devices, such as particle accelerators and gyrotrons, has gained increased interest. In this paper we focus on the effect disorder has on the resonant frequency and peak electric field in the defect site of a 2D PC structure. For disorders up to a maximum of 15% variation in position and radius, we found that disorder applied to the inner-most rods surrounding the defect site dominates in determining the peak field and resonant frequency of the structure. We also show that small disorder ($\sim 1\%$) can lead to an increase in peak field in certain cases. We find increasing levels of disorder lead to a decreasing average peak field for all structures. Whereas the mean resonant frequency remains constant for increasing disorder while the standard deviation increases. We then develop an understanding for this behaviour in terms of frequency detuning and mode confinement.

I. INTRODUCTION

High-power ElectroMagnetic (EM) technologies such as klystrons and particle accelerators utilise the interaction between charged particles and conductive structures to either store energy in EM fields or to extract energy from EM fields.

Conventional technologies used in particle accelerators suffer from a number of issues. These include parasitic effects such as wakefields¹, created by the radiation from particles passing through accelerator structures that can excite high order modes (HOMs) causing instabilities. There is also the need to improve intensity, stability, delivery and energy of the beam, driven by the need to reduce the size of accelerators due to practical and economic constraints.

Current accelerators generally operate at frequencies below 3 GHz. At higher frequencies (10's of GHz) accelerators can operate with reduced stored energy and power consumption². This has the advantage of higher average power capability and higher efficiency of operation^{2,3}. However, operating conventional technology at higher frequencies exacerbates the problems caused by wakefields, which scale with the cube of the frequency⁴, and hamper the use of conventional structures at higher frequencies.

Extensive research on novel acceleration techniques to address these issues⁵⁻⁸ is underway. One possible solution, which is gaining increased interest, is the use of Photonic Band Gap (PBG) structures⁹⁻¹¹. These structures have attracted considerable interest for their ability to confine, manipulate, guide and inhibit light¹²⁻¹⁶. For example: Smirnova *et al.*¹¹ have successfully demonstrated the acceleration of an electron beam at 17 GHz using a PBG structure; Sirigiri in¹⁷ demonstrated a gyrotron oscillator using a PBG structure; Smirnov and Yu have a design using PBG structures for a multibeam Klystron¹⁸; and we are considering applications of Crab-PBG structures.

Conventional high-power microwave technologies generally use either standing-wave or traveling-wave struc-

tures. Standing-wave structures (resonant cavities) establish standing electromagnetic waves with the electric field along the direction of propagation. The frequency of a resonant cavity is very dependent upon geometry, in particular the radius, where the dimensions scale inversely with frequency¹⁹.

In PBG structures, resonant cavities can be formed by creating a defect in the periodic lattice. This allows an EM mode to exist inside the band gap which cannot propagate through the lattice, localizing the mode at the defect site. This frequency dependence makes it possible to create a structure where a specific mode is confined, but the HOMs which wakefields excite are not present at the defect site. These HOMs propagate away from the defect site through the PBG lattice. This ability of PBG structures to inhibit light offers a unique way to suppress wakefields, enabling operation of accelerators at frequencies of 10's of GHz.

Any physically realizable structure will inherently have some degree of disorder, therefore it is important to understand the effect this disorder will have on the properties of the structure. Previous papers have extensively studied the effect that disorder in PBG structures has on transmission and reflection properties²⁰⁻²². The majority of this previous work has focused on disorder in bulk photonic crystals with very few looking at cavity resonators, and only one paper to date examining the effect of disorder on resonant frequency²³. Zhu *et al.*²³ focused on applying white noise to the whole photonic crystal structure, looking at the resonant frequency and the quality factor of the cavity, only considering the case of a purely dielectric structure.

In this paper we extend this work by investigating the effects of disorder on the resonant frequency and the EM field distribution of a metallic photonic band gap cavity resonator. We do this by moving specific scatterers of the photonic crystal. We have focused on metallic structures as they are more commonly used than dielectric resonators in high power EM applications. Dielectric structures have specific issues, such as surface charging, high-losses, breakdown and multipactor discharges that

inhibit performance.

II. METHODOLOGY

The most commonly used type of PBG lattice in high power microwave applications is the 2-Dimensional (2D) triangular lattice of metal rods with separation \mathbf{a} and radius \mathbf{r} . In this paper the central rod is removed to create a defect in the band gap, forming the site of the cavity resonator. Figure 1 shows the ideal PBG structure, considered in this paper. The base structure is the ideal case consisting of rods with identical separation $\mathbf{a} = 0.0124\text{m}$ and identical radius $\mathbf{r} = 0.00186\text{m}$. The resonator formed by removing the central rod has a resonant frequency (or ‘base frequency’ f_0) of 9.4072 GHz. Any changes in the rod separation or radius are made in relation to this base structure. As a comparison, the base structure was also modeled using MAFIA, with an agreement with the COMSOL determined base frequency to the 8th decimal place. Any higher order modes are restricted to the edges of the structure.

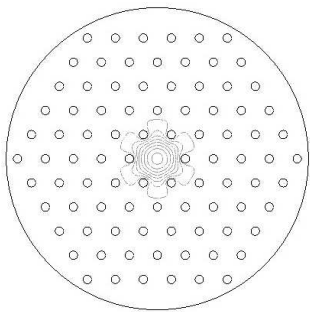


FIG. 1: Ideal Base PBG Structure, consisting of Perfectly Electrically Conducting (PEC) rods surrounded by a PEC wall. Removal of the central rod localises a single EM mode at $f_0 = 9.4072\text{ GHz}$. All rods have a radius of 0.00186m and a separation of 0.0124m . The contours show the extent of the electric field of the mode.

Disorder was introduced to the structure by adding \pm a random number between 0-15%, 0-10%, 0-5%, or 0-1%, of the initial parameters \mathbf{a} and \mathbf{r} , to each individual rod. Depending which parameter this disorder is applied to, it has the effect of altering the position, radius or ‘both’, meaning a combination of both the position and radius, of each rod. The random number used was taken from a uniform distribution pseudo-random number generator. This effectively introduces a white noise error to the dimensions of the PBG structure.

Focusing on the example of the position of the rods with a 10% disorder applied to the whole structure, 30 different random configurations of the structure were generated. Each disordered structure was processed to find the resonant frequency and the peak electric field at the centre of the structure. The results from these 30 disordered structures were then averaged to give a mean

value of the resonant frequency and peak electric field for a structure with 10% disorder to the position of the rods. This process was then repeated for different percentage disorders applied to the position, radius or both. The analysis was then extended to more specific cases.

The resonant frequency of each PBG structure was calculated using the commercially available finite element package COMSOL Multiphysics. This software was used to find the eigenmodes of each structure, using an automatic mesh refining technique to ensure accuracy of the solution.

To determine the peak electric field, simulations were performed using a finite-difference time-domain (FDTD) method²⁴, with a freely available software package with subpixel smoothing for increased accuracy²⁵. Each disordered structure was excited with a point source of frequency 9.4072 GHz (the resonant frequency of the ideal structure). For each disordered structure the source was allowed to run for the 20 RF cycles, it was then switched off and the simulation was then left to run for 20 RF cycles. The electric field was then recorded for next 20 RF cycles. Analysis of the recorded peak electric field at each cycle shows a small variation due to the finite spatial and temporal resolution of the FDTD technique. By increasing the resolution, this variation was reduced to between 1-2% between all 20 cycles. The value of the peak electric field taken for each disordered structure is the mean value of the peak field during the last set of 20 RF cycles.

III. RESONANT FREQUENCY

To explore how disorder effects the resonant frequency of the PBG structure the finite element package COMSOL Multiphysics was used to determine the eigenmodes of various disordered PBG structures. Results are compared to the resonant frequency of the base structure of figure 1.

Initially, we considered disorder applied to position, radius and both for all rods in the structure. Disorder of 10%, 5%, and 1% of the initial parameters \mathbf{a} and \mathbf{r} was investigated. For each case, 30 disordered structures were considered and averaged as outlined above. The effects of a random disorder applied to the whole structure are shown in figure 2. The resonant frequency is plotted against the percentage disorder for position, radius and both. The mean value of the resonant frequency for each disorder (1%, 5%, 10%) is shown by the plotted marker. The solid line indicates the resonant frequency of the ideal base structure, f_0 , and the vertical bars show one standard deviation from the mean.

The results show for disorders of 1% and 5%, the mean only varies slightly from the base frequency. We note that for a 10% disorder, the variation of the mean from the base frequency is about 1%. In terms of percentage disorder, separation has a larger effect than radius. While the effect on the resonant frequency of applying disorder

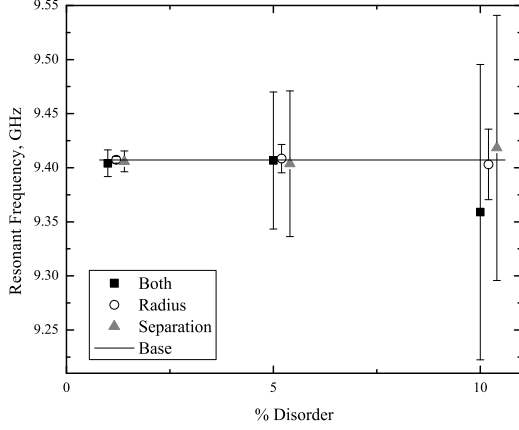


FIG. 2: The effect of disorder in radius, separation and both applied to the whole structure. The fundamental resonant frequency is shown against disorder magnitude for 1%, 5%, and 10% disorder. For each case of radius, separation and both, the data is plotted laterally displaced for clarity. The markers show the mean resonant frequency averaged over 30 structures. The vertical error bars show one standard deviation from the mean. The horizontal line indicates the base frequency.

der to both, can be seen as approximately equal to the sum of the separate variations in position and radius. In terms of absolute variation, where a 5% variation in radius (~ 0.1 mm) is approximately equal to a 1% variation in position (~ 0.1 mm), we can see that the effect on the resonant frequency is approximately equal in both cases.

Although the mean resonant frequency remains fairly constant and close to that of the base structure, we note that for some structures increasing disorder causes the resonant frequency to significantly deviate from the base value. To understand this behaviour we analysed the effect on the resonant frequency of altering the position and radius of individual rods. Starting with the ring of the innermost rods, closest to the centre of the structure (labeled ring 1), each rod is systematically moved by 10%, 5%, and 1% of its' initial separation a into and out from the centre. This process was then applied to the rods in the second and third rings of rods from the centre. It was finally extended to the rods of the outer rings but was found to have a negligible effect (less than 0.001%).

Figure 3 shows the results of this examination. We can immediately see that the effect of moving the rods of ring 1, dominates over the other rings. As the rods are moved into the centre the resonant frequency increases, while moving the rods away from the centre results in a decrease in the resonant frequency. This is the case for the rods of all rings, although the further out the ring, the lesser the effect it has upon the resonant frequency. This behaviour can be understood in terms of perturbation to the cavity geometry as discussed in reference 19.

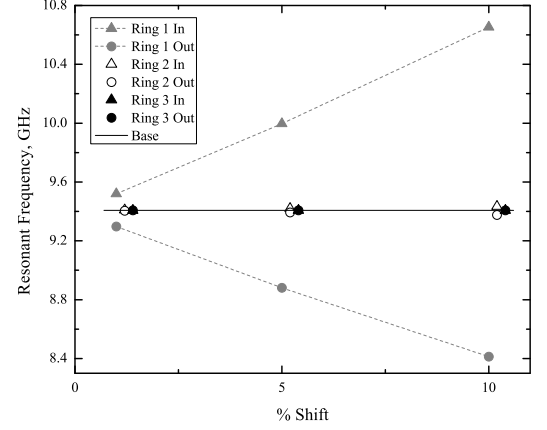


FIG. 3: Considering the rods in the base structure as arranged in rings of rods around the defect region, the innermost ring labeled ring 1. This graph shows the fundamental resonant frequency achieved by moving all rods in a specific ring in and out by various percentages relative to the base structure. The horizontal line indicates the base frequency.

Inward perturbations raise the resonant frequency and outward perturbations decrease the resonant frequency, so as the rods of ring 1 are moved in towards the centre of the structure the volume of the defect region decreases, this naturally results in a higher resonant frequency of the structure. As the rods are moved out, away from the centre, the larger volume results in the observed frequency decrease. As the size of the perturbation increases so does the relative shift in resonant frequency.

With this in mind, when considering disorder applied around the defect region, as disorder increases the spread in resonant frequencies of the structure increases, as seen in figure 2. This argument would imply that the mean frequency should be equal to the resonant frequency of the base structure. To develop this argument further a random disorder introduced into each individual ring was investigated and we can start to consider how disorder in each ring of rods contributes to the performance of the entire structure. This analysis was done by introducing disorder separately to each ring in turn. 10%, 5%, and 1% disorder is introduced to rod position, radius and both. In each case all other rods are kept in the base configuration. The results of this analysis are shown in figure 4. Even with a 10% disorder the effects of this disorder in rings three, four and five were found to be negligible, so for clarity are not presented here. For disorders of 1% and 5% the mean only varies slightly from the base structure, whereas for a 10% disorder the variation of the mean from the base structure is quite marked. This is in agreement with the results shown in figure 2 but conflict with the arguments surrounding figure 3. To investigate this further we examined the frequency of each structure with 10% disorder to both. We found that out of the

30 samples, two had particularly low values. To investigate this, we increased the number of structures at 10% disorder to both, from 30 to 40. The mean value for 40 structures is shown in figure 4 by the ‘cross’ marker. We see that this mean is closer to the base frequency, which suggests that the variation of the mean from the base frequency is due to the limited number of samples.

We also examined the effect that disorder has on the position and frequency of higher order modes of the structure. We found that even at 10% disorder the frequencies and positions of the higher order modes were unaffected.

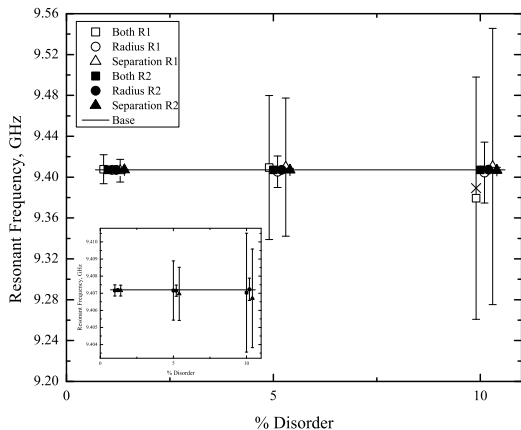


FIG. 4: The effect that disorder of radius and separation in each ring of rods has on resonant frequency. The markers show the mean resonant frequency taken over 30 structures, with error bars showing one standard deviation from the mean. The horizontal line shows the resonant frequency of the base structure. The extra cross marker at the 10% level of disorder is the mean frequency found by increasing the number of structures from 30 to 40. The insert shows the results of disorder in ring 2 to give a clearer picture of the behaviour.

IV. PEAK FIELD

To examine how disorder effects the peak electric field in PBG structures, the FDTD technique outlined in section II was used to excite electric fields at the centre of the defect site of PBG structures with various, random or specific, rod displacements and radii variations. The results were then compared to the base structure. The peak electric field for the base structure (the ‘base field’) is 26.749 V. In all cases the EM source at the fundamental frequency 9.4072 GHz is used.

As in the case of the resonant frequency, firstly we considered disorder applied to the whole structure in the cases of position, radius and both. Disorder of 15%, 10%, 5%, and 1% of the initial parameters \mathbf{a} and \mathbf{r} was investigated. For each case 30 disordered structures were

considered and averaged as outlined previously. The numerical results of these simulations are shown in figure 5. The peak E_z component of the electric field is plotted against the percentage disorder for position, radius, and both. The solid horizontal line indicates the base field. The mean value of E_z for each disorder (1%, 5%, 10%, 15%) is shown by the plotted marker. The vertical bars show one standard deviation from the mean.

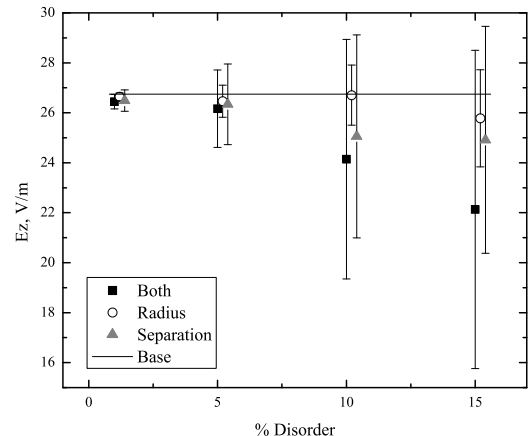


FIG. 5: The effect of disorder in radius, separation and both applied to the whole structure. The peak field at the defect site is shown against disorder magnitude for 1%, 5%, 10% and 15% disorder. For each case of radius, separation and both, the data is plotted laterally displaced for clarity. The markers show the mean peak field averaged over 30 structures. The vertical error bars show one standard deviation from the mean. The horizontal line indicates the base field.

As one would expect, increasing disorder of the structure causes the mean peak field to decrease. Again, as in the case of the resonant frequency, in terms of percentage disorder, separation has a larger effect than radius. While the effect on the peak field of applying disorder to both, can be seen as approximately equal to the sum of the separate variations in position and radius. In terms of absolute variation, we can see that the effect on the peak field is approximately equal in both cases.

Although overall the trend of the mean peak field is to decrease with increasing disorder, there are some structures that produce results showing the opposite behaviour. Looking at moving the individual rods of the rings into and out from the centre sheds some light on this behaviour. Again, each rod in a ring of rods, is systematically moved by 10%, 5%, and 1% of its initial separation \mathbf{a} into and out from the centre while all other rods remain in the base configuration. These results are shown in figure 6. Moving the rods of ring 1 has the greatest effect on the peak field, as in the case of the resonant frequency results. The effects of moving ring 1 can be understood in terms of frequency detuning and mode concentration¹². Movement of the rods

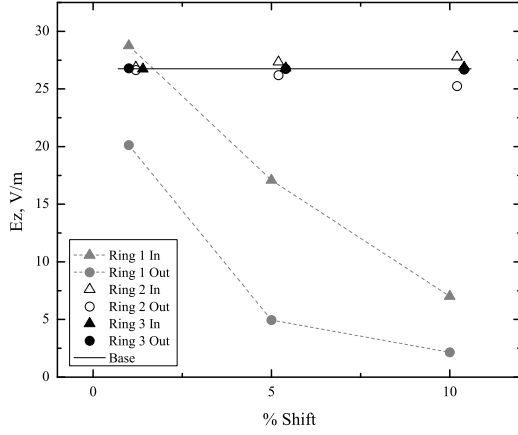


FIG. 6: Considering the rods in the base structure as arranged in rings of rods around the defect region, the innermost ring labeled ring 1. This graph shows the peak field achieved by moving all rods in a specific ring in and out by various percentages relative to the base structure. The horizontal line indicates the base field.

by 5%, 10%, and 15%, either in or out, is so large that the resonant frequency of the defect region is either increased or decreased by a significant amount (cf. figure 3). The resonant frequency of the structure now shifts to a frequency f_s . This shift in the resonant frequency of the structure means that the frequency of the excitation source at f_0 is so far from the resonant frequency, f_s , of the structure, that the EM field poorly couples to the cavity. This leads to a decrease in energy stored in the excited field compared to the base structure. In the case of a 1% movement of the rods in or out, the shift in the resonant frequency of the cavity (as shown in figure 3) is small enough that the excitation source still couples strongly to the resonator. Using the FDTD technique, we analysed the field energy in the defect region. In all three cases (base, move rods-in, move rods-out) the total energy in the volume of the defect region is constant. Moving the rods in and out changes the volume over which this energy is distributed. The energy stored in the EM field is given by $(E \cdot D + |H|)/2$, where E and H are the electric and magnetic field components and D the electric displacement field. So in order to maintain a constant energy, a reduction in the volume results in an increase in EM field magnitude. Likewise, an increase in the volume results in a decrease of the EM field magnitude. This same interpretation can be applied to moving the rods of ring 2, although to a much lesser extent.

Considering how disorder within each ring effects the peak field, 10%, 5%, and 1% disorder was introduced to rod position, radius and both, introducing disorder separately to each ring in turn. Again, in each case all other rods are kept in the base configuration. The results of this FDTD analysis are shown in figure 7. The effects

of disorder in rings three, four and five were found to be negligible, so for clarity are not presented.

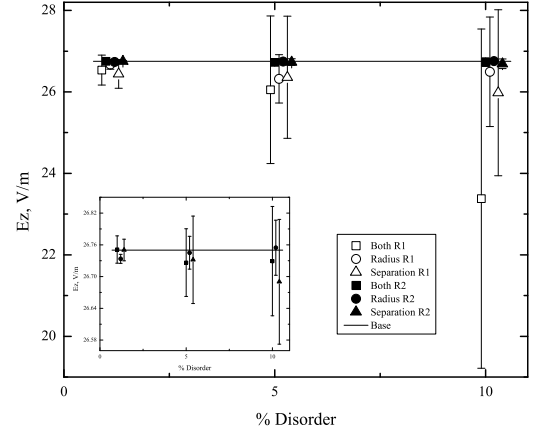


FIG. 7: The effect that disorder of radius and separation in each ring of rods has on peak field. The markers show the mean peak field taken over 30 structures, with error bars showing one standard deviation from the mean. The horizontal line shows the peak field of the base structure. The insert shows the results of disorder in ring 2 to give a clearer picture of the behaviour.

Note that the effect of disorder of the inner ring (ring 1) dominates over the effects of the other rings of rods. In terms of percentage disorder, separation has a larger effect than radius, and in terms of absolute variation, separation and radius have about equal effect.

As seen in figure 5, the mean peak field decreases with increasing disorder when disorder is introduced to the whole structure. We see the same behaviour reproduced in figure 7 when the disorder is introduced into ring 1 only. The magnitude of displacement and direction is determined using a pseudo-random number generator with a uniform distribution. As percentage of total disorder increases, a greater number of rods are subject to a larger change in their position or radius. This leads to an increase in the number of structures with a lower value in the peak field. Comparing figures 5 and 7 we see the behaviour due to disorder in the entire structure is effectively defined solely by disorder in ring 1.

An understanding of how disorder in position and radius interact can be found by examining the distribution of the peak field values for a large number of structures. Figure 8 shows the distribution of the peak field from 300 different structures of 5% disorder to both. The inset graphs show the distribution of peak field from 300 different structures of 5% disorder to position and radius separately. The distributions for disorder in either radius or separation are slightly skewed, with the peak field of the base structure higher than the mean peak field of the disordered structure. This can be understood by considering the arguments presented above concerning the

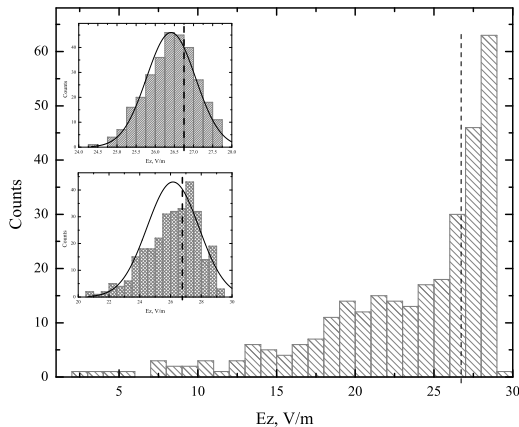


FIG. 8: Distribution of peak field value for 300 structures. The main curve shows the distribution for 5% disorder to both radius and position. The inset curves show; Top, 5% disorder to radius and bottom, 5% disorder to position. The dotted line shows the base structure peak field.

effect on the peak field of moving the rods in and out. Generally, more structures are created with rod position or radius configurations that result in a lower peak field than the base. Hence, shifts of greater than 1% inwards will result in detuning of the cavity and therefore a lower field. By increasing the magnitude of disorder these effects on average occur with increased probability, shifting the mean peak field for the disordered structure to a lower value. The distribution for disorder in both, shown in figure 8, becomes highly skewed. We can see that, although for the majority of structures the peak field is lower than the base peak field, for approximately 30% of the samples the peak field is greater than the base peak field. This can be seen from the distribution curves for disorder in position or radius alone. There are structures where the combination of radius and position result in a 1% (or less) effective inward reduction of modal volume, leading to a slight increase in peak field. The combination of disorder is more likely to increase the modal volume or detune the cavity, therefore decreasing the peak field. This yields a skewed distribution where the maximum peak field for a structure is 29 V/m and a minimum peak field is close to zero. The mean peak field of the structures decrease almost linearly with increasing magnitude of disorder.

Considering the distribution of the results of the resonant frequency analysis and comparing it to those of the peak field discussed above, the distribution for a disorder in both is less skewed and has a normal distribution.

Using the FDTD technique we also examined the quality factor of the disordered PBG resonant cavities. The quality factor is the ratio of the energy in the cavity to the energy lost. We found that the introduction of disorder has a small effect on the quality factor, in agreement with the work of Zhu *et al.*²³

V. CONCLUSIONS

For disorders up to a maximum of 15% variation in position and radius, we found that disorder applied to the inner-most rods surrounding the defect site dominates in determining the peak field and resonant frequency of the structure. Disorder in all other rods, up to a maximum of 15%, has a small effect on PBG performance. Although disorder in the outer rods has a small effect on performance of the resonant structures, the presence of the outer rods are required for mode confinement. Smirnova *et al.*¹⁰ have shown that reducing the number of rings of rods in a PBG structure has an adverse effect on the ability of the structure to propagate higher order modes away from the defect site and reduces the ability of structure to localize the mode of interest. In terms of absolute variation, disorder leads to a larger variation in peak field than in resonant frequency. A 5% disorder to both (radius and position) leads to a maximum 0.5% variation in resonant frequency (40 MHz) and a maximum 5% variation in peak electric field (1 V/m). This behaviour is predominantly determined by the disorder in ring 1 of the rods only. Examination of the results show that a 1% disorder in ring 1 has a greater effect than a 10% disorder in all other rods. We have also shown that a small disorder ($\sim 1\%$) of the innermost ring of rods can actually lead to an increase in peak field, by decreasing the volume over which the energy of the EM field is distributed. Increasing disorder leads to a decrease in the structures average peak field, where as the mean resonant frequency remains constant with an increasing standard deviation. We show how this behaviour is dependant on varying the radius and position of the rods, which detune the cavity.

We have found that randomly introduced disorder and systematically moving individual rods, results in the ability to ‘tune’ the PBG structure and have found it possible to increase peak field by approximately 10%. This could prove beneficial in the design of PBG based accelerating structure where there is a requirement to maximize the peak electric field and thereby maximize accelerating gradient.

In terms of structure fabrication, a maximum error in the inner most ring of rods of 1% in separation, 5% in radius, and less than 10% disorder in all outer rods, leads to an average resonant frequency equal to the ideal structure (9.4072 GHz) with a maximum variation of 0.2% (20 MHz), and maximum variation of 0.5 V/m in the peak field. To achieve this for the structure considered in this paper requires fabrication of the rods with a radius variation of $150\ \mu\text{m}$, and a separation variation of $100\ \mu\text{m}$ for the inner most ring of rods. This level of accuracy in fabrication although difficult is within the capability of modern fabrication facilities.

VI. ACKNOWLEDGEMENTS

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